K02 ACCELERATION

SPH4U

CH 1 (THE BIG PICTURE)

- the linear motion of objects in horizontal, vertical, and inclined planes
- the motion of a projectile in terms of components of its motion
- objects moving in two dimensions
- predict the motion of an object
- technological devices based on the concepts and principles of projectile motion

EQUATIONS

- Instantaneous Acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

- Kinematic Equations:

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\Delta \vec{d} = \vec{v}_i \Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$

$$\Delta \vec{d} = \frac{1}{2}\left(\vec{v}_i + \vec{v}_f\right)\Delta t$$

$$v_f^2 = v_i^2 + 2a\Delta d$$

$$\Delta \vec{d} = \vec{v}_f \Delta t - \frac{1}{2}\vec{a}(\Delta t)^2$$

ACCELERATION AND AVERAGE ACCELERATION IN ONE DIMENSION

- **Acceleration ($\vec{a}$) [m/s^2]:** rate of change of velocity
- **Average Acceleration ($\vec{a}_{av}$):** change in velocity divided by the time interval for that change

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

- **Instantaneous Acceleration:** acceleration at a particular instant

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

NOTE:

- **Jerk:** the rate of change of acceleration

SAMPLE PROBLEM 1

A racing car accelerates from rest to 96 km/h [W] in 4.1 s. Determine the average acceleration of the car.

SAMPLE PROBLEM 1 – SOLUTIONS

$\vec{v}_i = 0.0 \text{ km/h}$

$\vec{v}_f = 96 \text{ km/h [W]}$

$\Delta t = 4.1 \text{ s}$

$\vec{a}_{av} = ?$

$$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$= \frac{96 \text{ km/h [W]} - 0.0 \text{ km/h}}{4.1 \text{ s}}$$

$$\vec{a}_{av} = 23 \text{ (km/h)/s [W]}$$

The average acceleration of the car is 23 (km/h)/s [W].

SAMPLE PROBLEM 2

A motorcyclist travelling at 23 m/s [N] applies the brakes, producing an average acceleration of 7.2 m/s^2 [S].

(a) What is the motorcyclist's velocity after 2.5 s?

(b) Show that the equation you used in (a) is dimensionally correct.

SAMPLE PROBLEM 2 – SOLUTIONS

(a) $\vec{v}_i = 23 \text{ m/s [N]}$

$\vec{a}_{av} = 7.2 \text{ m/s}^2 \text{ [S]} = -7.2 \text{ m/s}^2 \text{ [N]}$

$\Delta t = 2.5 \text{ s}$

$\vec{v}_f = ?$

From the equation $\vec{a}_{av} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$,

$$\begin{aligned}\vec{v}_f &= \vec{v}_i + \vec{a}_{av}\Delta t \\ &= 23 \text{ m/s [N]} + (-7.2 \text{ m/s}^2 \text{ [N]})(2.5 \text{ s}) \\ &= 23 \text{ m/s [N]} - 18 \text{ m/s [N]} \\ \vec{v}_f &= 5 \text{ m/s [N]}\end{aligned}$$

The motorcyclist's final velocity is 5 m/s [N].

SAMPLE PROBLEM 2 – SOLUTIONS

(b) We can use a question mark above an equal sign $\left(\stackrel{?}{=}\right)$ to indicate that we are checking to see if the dimensions on the two sides of the equation are the same.

$$\vec{v}_f \stackrel{?}{=} \vec{v}_i + \vec{a}_{av}\Delta t$$

$$\frac{L}{T} \stackrel{?}{=} \frac{L}{T} + \left(\frac{L}{T^2}\right)T$$

$$\frac{L}{T} \stackrel{?}{=} \frac{L}{T} + \frac{L}{T}$$

The dimension on the left side of the equation is equal to the dimension on the right side of the equation.

GRAPHING MOTION WITH CONSTANT ACCELERATION

- When data is curved, we need to find the instantaneous velocities
- Recall: tangents

Table 1 Position-Time Data

t (s)	$\vec{d}$ (m [E])
0	0
2.0	8.0
4.0	32
6.0	72
8.0	128

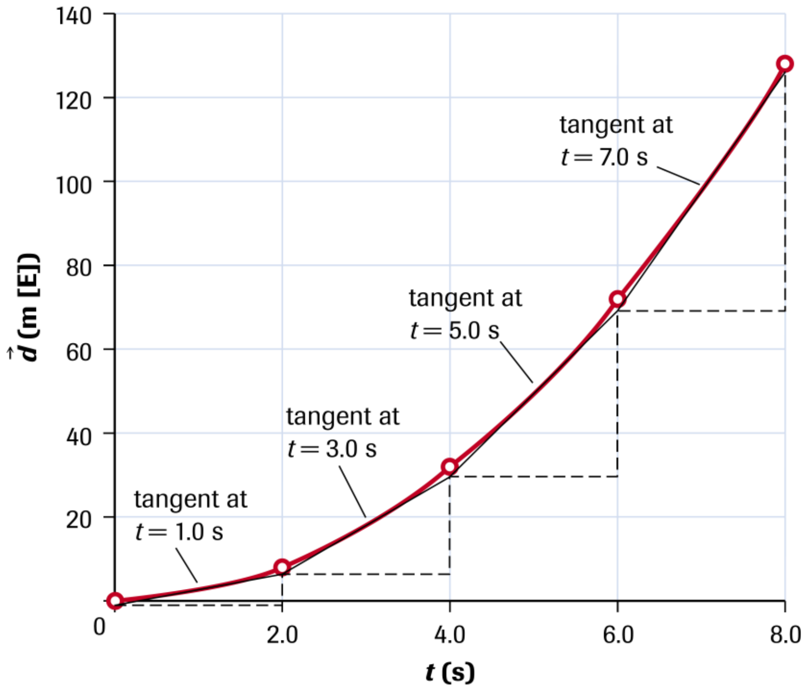


Figure 3
On this position-time graph of the boat's motion, the tangents at four different instants yield the instantaneous velocities at those instants.

GRAPHING MOTION WITH CONSTANT ACCELERATION

- From the instantaneous velocities calculated using the previous graph, we get the following:

Table 2 Velocity-Time Data

t (s)	$\vec{v}$ (m/s [E])
0	0
1.0	4.0
3.0	12
5.0	20
7.0	28

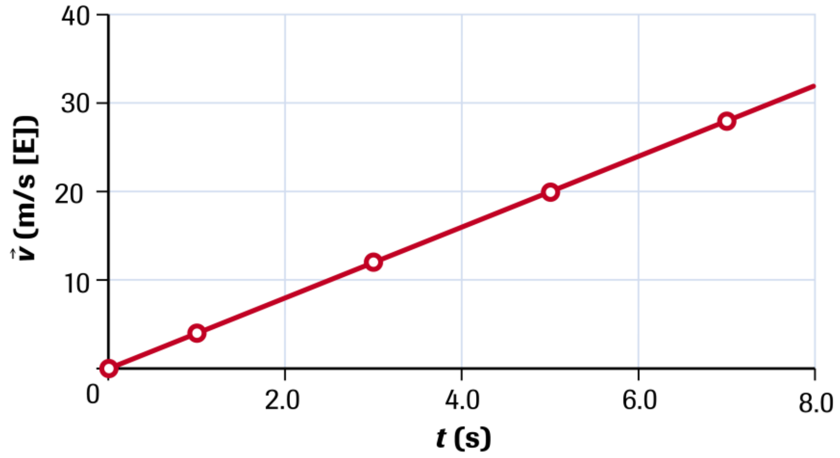


Figure 4
The velocity-time graph of motion with constant acceleration is a straight line. How would you find the instantaneous acceleration, average acceleration, and displacement of the boat from this graph?

GRAPHING MOTION WITH CONSTANT ACCELERATION

- Since the velocity-time graph is a straight line, we can use the slope to find acceleration

$$\vec{a} = \vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$$

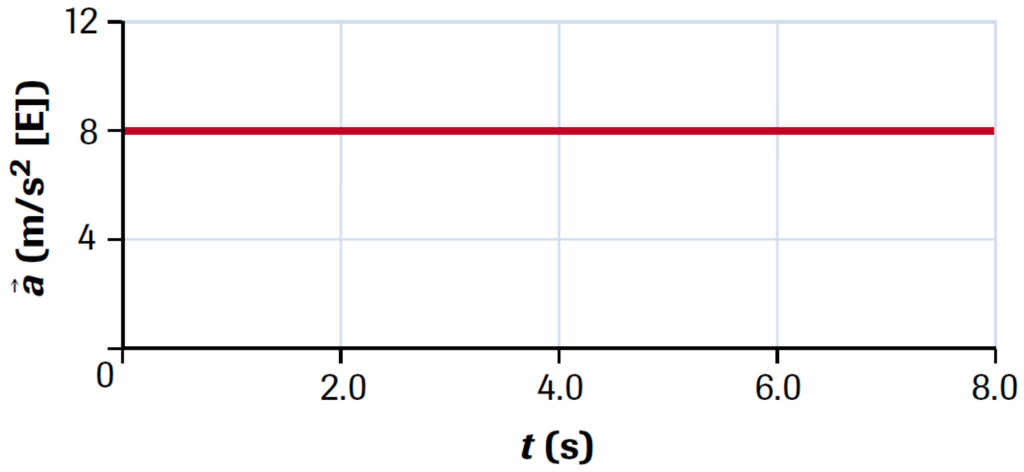


Figure 5
The acceleration-time graph of motion with constant acceleration is a straight horizontal line. How would you find the change in velocity over a given time interval from this graph?

SAMPLE PROBLEM 3

Figure 6 is the acceleration-time graph of a car accelerating through its first three gears. Assume that the initial velocity is zero.

(a) Use the information in the graph to determine the final velocity in each gear. Draw the corresponding velocity-time graph.

(b) From the velocity-time graph, determine the car's displacement from its initial position after 5.0 s.

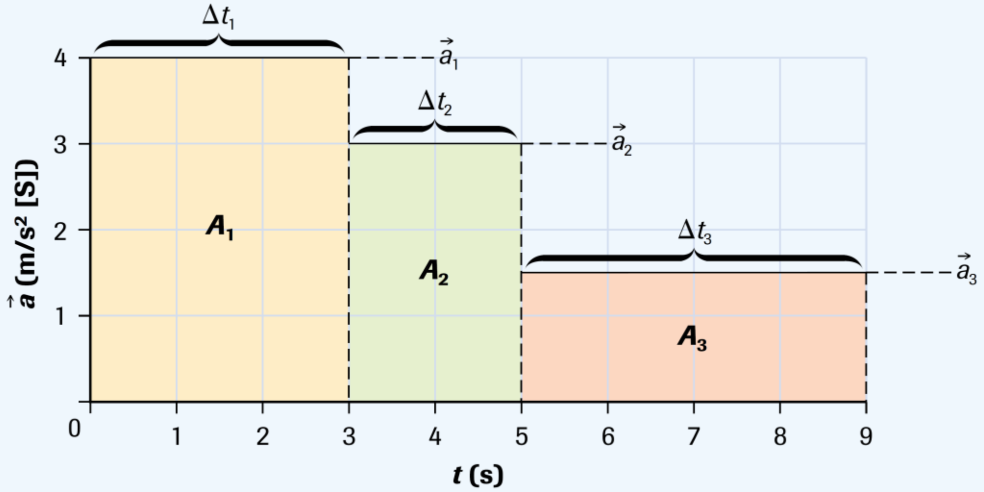


Figure 6
Acceleration-time graph

SAMPLE PROBLEM 3 – SOLUTIONS

(a) The area beneath each segment of the acceleration-time plot is the change in velocity during that time interval.

$$A_1 = \vec{a}_1 \Delta t_1 = (4.0 \text{ m/s}^2 \text{ [S]})(3.0 \text{ s})$$
$$A_1 = 12 \text{ m/s [S]}$$

$$A_2 = \vec{a}_2 \Delta t_2 = (3.0 \text{ m/s}^2 \text{ [S]})(2.0 \text{ s})$$
$$A_2 = 6.0 \text{ m/s [S]}$$

$$A_3 = \vec{a}_3 \Delta t_3 = (1.5 \text{ m/s}^2 \text{ [S]})(4.0 \text{ s})$$
$$A_3 = 6.0 \text{ m/s [S]}$$

$$A_{total} = A_1 + A_2 + A_3 = 12 \text{ m/s} + 6.0 \text{ m/s} + 6.0 \text{ m/s}$$
$$A_{total} = 24 \text{ m/s}$$

The initial velocity is $\vec{v}_1 = 0.0$ m/s. The final velocity in first gear is $\vec{v}_2 = 12$ m/s [S], in second gear is $\vec{v}_3 = 18$ m/s [S], and in third gear is $\vec{v}_4 = 24$ m/s [S].

Figure 7 is the corresponding velocity-time graph.

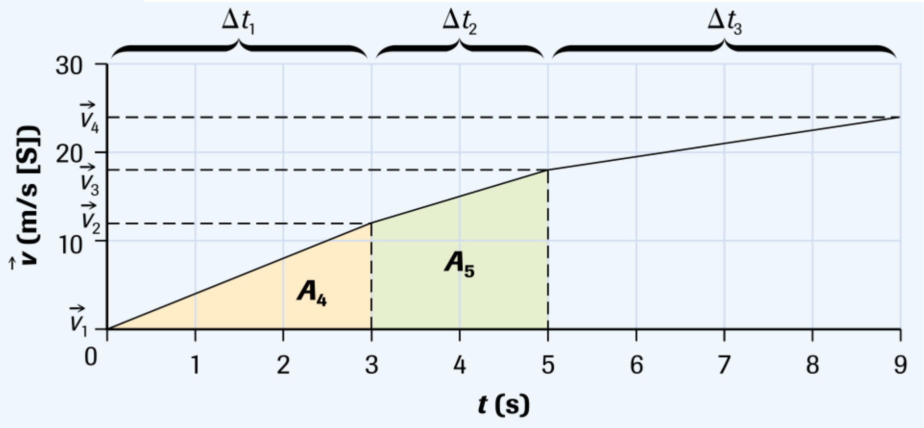


Figure 7
Velocity-time graph

SAMPLE PROBLEM 3 – SOLUTIONS

(b) The area beneath each line on the velocity-time graph yields the change in position during that time interval.

$$A_4 = \frac{1}{2}\left(\vec{v}_2 - \vec{v}_1\right)(\Delta t_1)$$

$$= \frac{1}{2}(12 \text{ m/s [S]})(3.0 \text{ s})$$

$$A_4 = 18 \text{ m [S]}$$

$$A_5 = \left(\vec{v}_2\right)(\Delta t_2) + \frac{1}{2}\left(\vec{v}_3 - \vec{v}_2\right)(\Delta t_2)$$

$$= (12 \text{ m/s [S]})(2.0 \text{ s}) + \frac{1}{2}(18 \text{ m/s [S]} - 12 \text{ m/s [S]})(2.0 \text{ s})$$

$$A_5 = 30 \text{ m [S]}$$

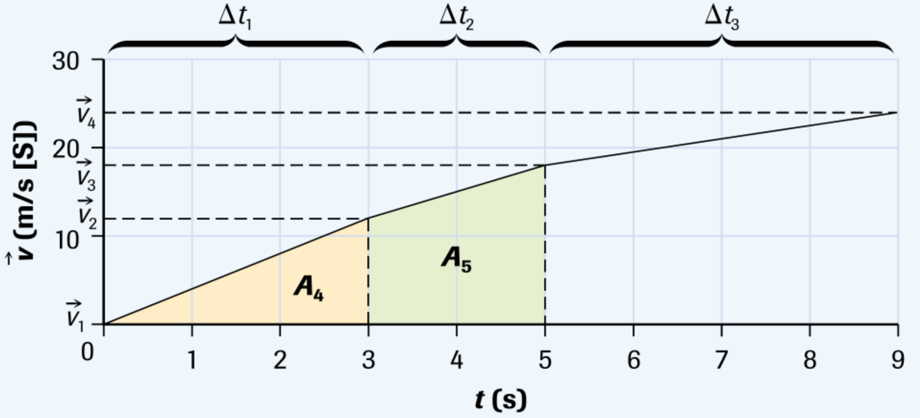


Figure 7
Velocity-time graph

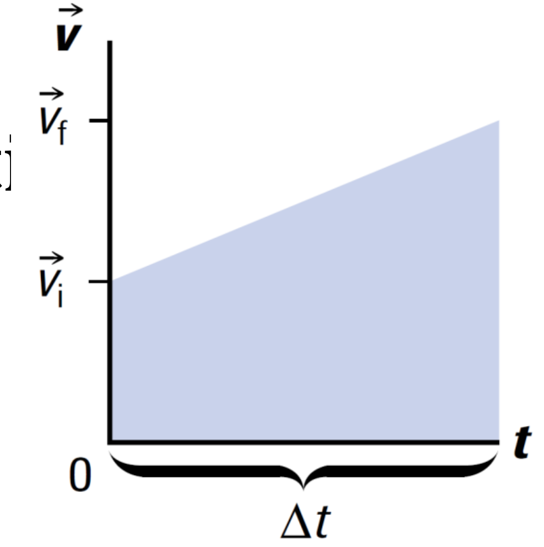
SOLVING CONSTANT ACCELERATION PROBLEMS

- When acceleration is constant,

$$\vec{a} = \vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

  - Note: this equation excludes displacement

- Displacement is the area under the velocity-ti graph, which is trapezoidal, so



$$\Delta\vec{d} = \tfrac{1}{2}(\vec{v}_i + \vec{v}_f)\Delta t$$

RECALL: KINEMATIC EQUATIONS

- Through substitution, we can derive the other kinematic ("suvat") equations

Variables Involved	General Equation	Variable Eliminated
$\vec{a}, \vec{v}_i, \vec{v}_f, \Delta t$	$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$	$\Delta\vec{d}$
$\vec{a}, \vec{v}_i, \Delta\vec{d}, \Delta t$	$\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$	$\vec{v}_f$
$\vec{v}_i, \vec{v}_f, \Delta\vec{d}, \Delta t$	$\Delta\vec{d} = \vec{v}_{av}\Delta t$ or $\Delta\vec{d} = \frac{1}{2}(\vec{v}_i + \vec{v}_f)\Delta t$	$\vec{a}$
$\vec{a}, \vec{v}_i, \vec{v}_f, \Delta\vec{d}$	$v_f^2 = v_i^2 + 2a\Delta d$	Δt
$\vec{a}, \vec{v}_f, \Delta\vec{d}, \Delta t$	$\Delta\vec{d} = \vec{v}_f\Delta t - \frac{1}{2}\vec{a}(\Delta t)^2$	$\vec{v}_i$

SAMPLE PROBLEM 4

A motorcyclist, travelling initially at 12 m/s [W], changes gears and speeds up for 3.5 s with a constant acceleration of 5.1 m/s^2 [W]. What is the motorcyclist's displacement over this time interval?

SAMPLE PROBLEM 4 – SOLUTIONS

$\vec{v}_i = 12 \text{ m/s [W]}$ $\quad\quad \Delta t = 3.5 \text{ s}$

$\vec{a} = 5.1 \text{ m/s}^2 \text{ [W]}$ $\quad\quad \Delta\vec{d} = ?$

$$\Delta\vec{d} = \vec{v}_i \Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$

$$= (12 \text{ m/s [W]})(3.5 \text{ s}) + \frac{1}{2}(5.1 \text{ m/s}^2 \text{ [W]})(3.5 \text{ s})^2$$

$$\Delta\vec{d} = 73 \text{ m [W]}$$

The motorcyclist's displacement is 73 m [W].

SAMPLE PROBLEM 5

A rocket launched vertically from rest reaches a velocity of 6.3×10^2 m/s [up] at an altitude of 4.7 km above the launch pad. Determine the rocket's acceleration, which is assumed constant, during this motion.

SAMPLE PROBLEM 5 – SOLUTIONS

$\vec{v}_i = 0$ m/s

$\vec{v}_f = 6.3 \times 10^2$ m/s [up]

$\Delta\vec{d} = 4.7$ km [up] $= 4.7 \times 10^3$ m [up]

$\vec{a} = ?$

We choose [up] as the positive direction. Since Δt is not given, we will use the equation

$$v_f^2 = v_i^2 + 2a\Delta d$$

$$v_f^2 = 2a\Delta d$$

$$a = \frac{v_f^2}{2\Delta d}$$

$$= \frac{(6.3 \times 10^2 \text{ m/s})^2}{2(4.7 \times 10^3 \text{ m})}$$

$$a = 42 \text{ m/s}^2$$

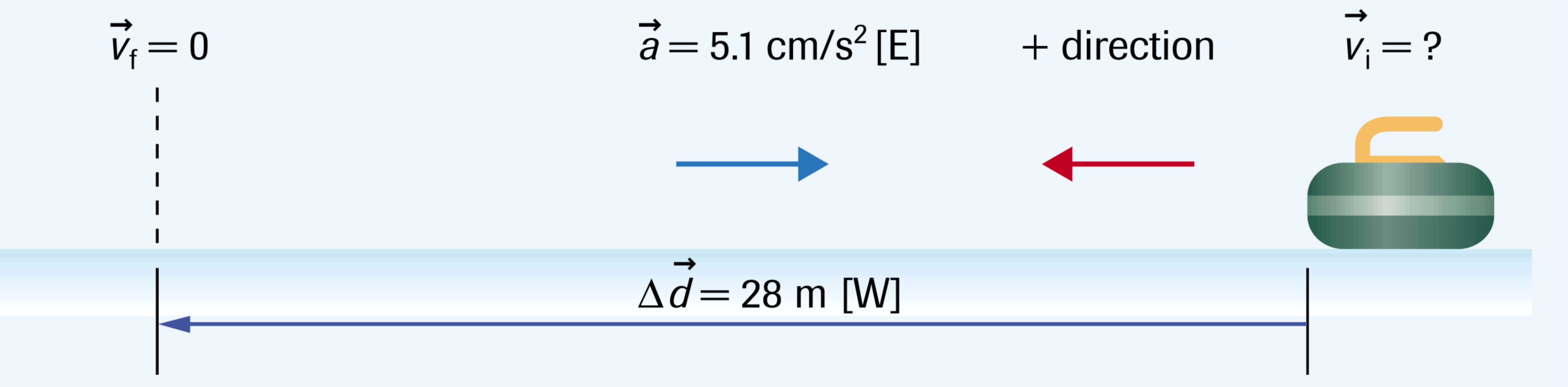
Since a is positive, the acceleration is 42 m/s^2 [up].

SAMPLE PROBLEM 6

A curling rock sliding on ice undergoes a constant acceleration of 5.1 cm/s^2 [E] as it travels 28 m [W] from its initial position before coming to rest. Determine (a) the initial velocity and (b) the time of travel.

SAMPLE PROBLEM 6 – SOLUTIONS

Figure 12 shows that the acceleration is opposite in direction to the motion of the rock and that the positive direction is chosen to be west.



SAMPLE PROBLEM 6 – SOLUTIONS

(a) $\Delta\vec{d} = 28 \text{ m [W]}$ $\quad \vec{a} = 5.1 \text{ cm/s}^2 \text{ [E]} = 0.051 \text{ m/s}^2 \text{ [E]} = -0.051 \text{ m/s}^2 \text{ [W]}$

$\vec{v}_f = 0 \text{ m/s}$ $\quad \Delta t = ?$

$\vec{v}_i = ?$

$$\begin{aligned}
v_f^2 &= v_i^2 + 2a\Delta d \\
0 &= v_i^2 + 2a\Delta d \\
v_i^2 &= -2a\Delta d \\
v_i &= \pm\sqrt{-2a\Delta d} \\
&= \pm\sqrt{-2(-0.051 \text{ m/s}^2)(28 \text{ m})} \\
v_i &= \pm 1.7 \text{ m/s}
\end{aligned}$$

The initial velocity is $v_i = 1.7 \text{ m/s [W]}$.

SAMPLE PROBLEM 6 – SOLUTIONS

(b) Any of the constant acceleration equations can be used to solve for Δt.

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\Delta t = \frac{\vec{v}_f - \vec{v}_i}{\vec{a}}$$

$$= \frac{0 - 1.7 \text{ m/s [W]}}{-0.051 \text{ m/s}^2 \text{ [W]}}$$

$$\Delta t = 33 \text{ s}$$

The time interval over which the curling rock slows down and stops is 33 s.

ACCELERATION IN TWO DIMENSIONS

- The same equations for one dimension work for two dimensions

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

- We break up the vectors into their components

$$a_{av,x} = \frac{\Delta v_x}{\Delta t} = \frac{v_{fx} - v_{ix}}{\Delta t} \qquad a_{av,y} = \frac{\Delta v_y}{\Delta t} = \frac{v_{fy} - v_{iy}}{\Delta t}$$

SAMPLE PROBLEM 7

The lawn mower in **Figure 13** takes 4.5 s to travel from A to B. What is its average acceleration?

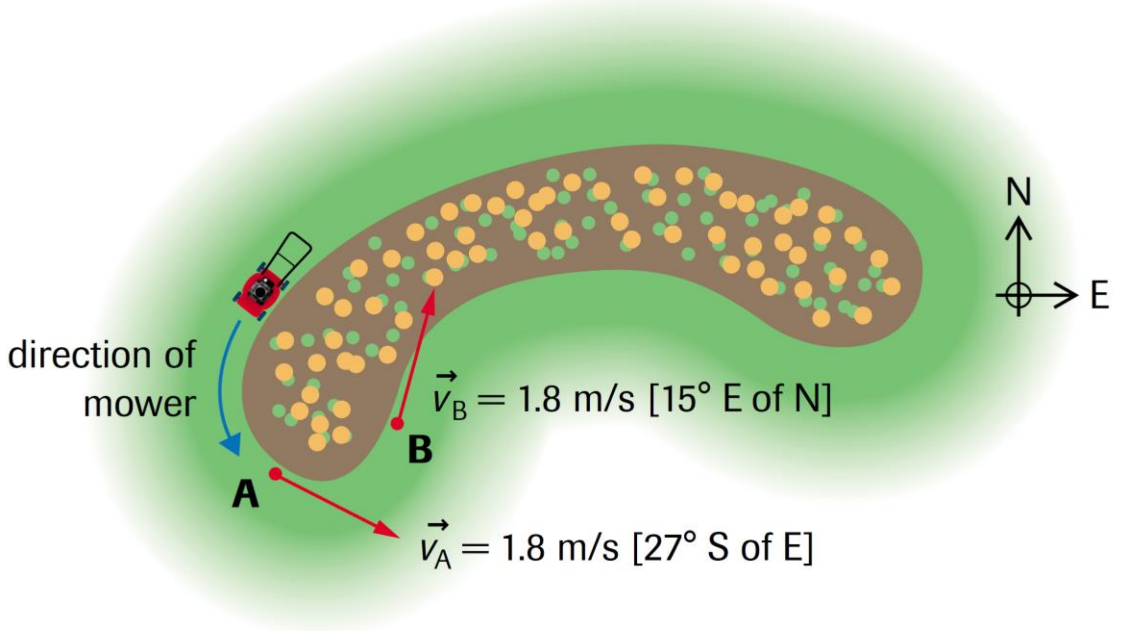


Figure 13
As the lawn mower follows the edge of the flowerbed at constant speed, it is accelerating: its direction of motion keeps changing.

SAMPLE PROBLEM 7 – SOLUTIONS

$\vec{v}_A = 1.8$ m/s [27° S of E] $\qquad \Delta t = 4.5$ s

$\vec{v}_B = 1.8$ m/s [15° E of N] $\qquad \vec{a}_{av} = ?$

We begin by finding $\Delta\vec{v}$, which is needed in the equation for average acceleration. In this case, we choose to work with vector components, although other methods could be used (such as the sine and cosine laws). The vector subtraction, $\Delta\vec{v} = \vec{v}_B + (-\vec{v}_A)$, is shown in **Figure 14**. Taking components:

$$\Delta v_x = v_{Bx} + (-v_{Ax})$$
$$= v_B \sin\theta + (-v_A \cos\beta)$$
$$= 1.8\text{ m/s}(\sin 15°) - 1.8\text{ m/s}(\cos 27°)$$
$$\Delta v_x = -1.1\text{ m/s}$$

$$\Delta v_y = v_{By} + (-v_{Ay})$$
$$= v_B \cos\theta + (-v_A \sin\beta)$$
$$= 1.8\text{ m/s}(\cos 15°) + 1.8\text{ m/s}(\sin 27°)$$
$$\Delta v_y = +2.6\text{ m/s}$$

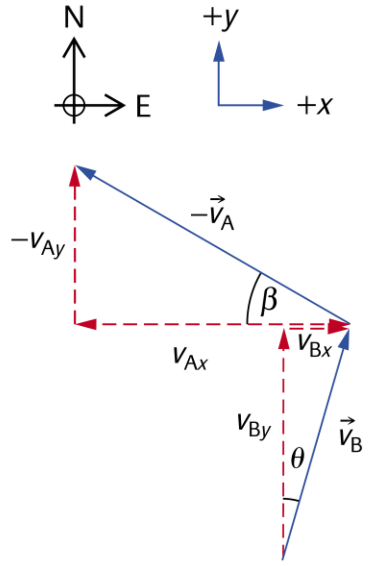


Figure 14
Determining the direction of the change in the velocity vector

SAMPLE PROBLEM 7 – SOLUTIONS

Using the law of Pythagoras,

$$|\Delta\vec{v}|^2 = |\Delta v_x|^2 + |\Delta v_y|^2$$
$$|\Delta\vec{v}|^2 = (1.1\ \text{m/s})^2 + (2.6\ \text{m/s})^2$$
$$|\Delta\vec{v}| = 2.8\ \text{m/s}$$

We now find the direction of the vector as shown in **Figure 15**:

$$\phi = \tan^{-1}\frac{1.1\ \text{m/s}}{2.6\ \text{m/s}}$$
$$\phi = 24°$$

The direction is [24° W of N].

To calculate the average acceleration:

$$\vec{a}_{\text{av}} = \frac{\Delta\vec{v}}{\Delta t}$$
$$= \frac{2.8\ \text{m/s [24° W of N]}}{4.5\ \text{s}}$$
$$\vec{a}_{\text{av}} = 0.62\ \text{m/s}^2\ \text{[24° W of N]}$$

The average acceleration is 0.62 m/s^2 [24° W of N].

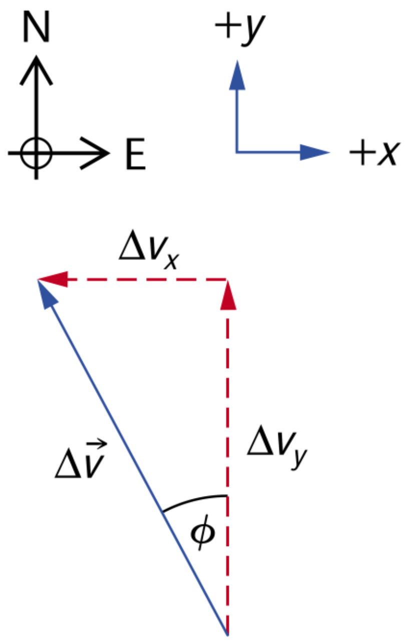


Figure 15
The velocities and their components

SUMMARY

- Average acceleration is the average rate of change of velocity.
- Instantaneous acceleration is the acceleration at a particular instant.
- The tangent technique can be used to determine the instantaneous velocity on a position-time graph of accelerated motion.
- The slope of the line on a velocity-time graph indicates the acceleration.
- The area under the line on an acceleration-time graph indicates the change in velocity.
- There are five variables involved in the mathematical analysis of motion with constant acceleration and there are five equations, each of which has four of the five variables.
- In two-dimensional motion, the average acceleration is found by using the vector subtraction v v_f v_i divided by the time interval t.

PRACTICE

Readings

- Section 1.2, pg 18

Questions

- pg 30 #1-18